UNIT 1 : NUMBER SEQUENCES

Learning Objectives

By the end of this chapter, you should be able to

- Recognise and use Fibonacci sequences
- Complete sequences in ordered pairs

General Sequences

Worked Examples: Complete the following sequences below

(a)	$2, 5, 8,$ ________ , ________ . **Solution**: Next two terms: 11 and 14 [Add *3* each time]	(b)	$10, 8, 6,$ ________ , ________ . **Solution**: Next two terms: 4 and 2 [Subtract *2* each time]
(c)	$4, 10, 16,$ ________ , ________ . **Solution**: Next two terms: 22 and 28 [Add *6* each time]	(d)	$20, 15, 10,$ ________ , ________ . **Solution**: Next two terms: 5 and 0 [Subtract *5* each time]
(e)	$1, 11, 21,$ ________ , ________ . **Solution**: Next two terms: 31 and 41 [Add *10* each time]	(f)	$-5, -12, -19,$ ________ , ________ . **Solution**: Next two terms: -26 and -33 [Subtract 7 each time]
(g)	$1, 2, 4,$ ________ , ________ . **Solution**: Next two terms: 8 and 16 [Multiply by *2* each time]	(h)	$-3, -9, -27,$ ________ , ________ . **Solution**: Next two terms: -81 and -243 [Multiply by *3* each time]
(i)	$-4, 8, -16,$ ________ , ________ . **Solution**: Next two terms: 32 and -64 [Multiply by -2 each time]	(j)	$1.5, 2.0, 2.5,$ ________ , ________ . **Solution**: Next two terms: 3.0 and 3.5 [Add 0.5 each time]
(k)	$-128, 64, -32,$ ________ , ________ . **Solution**: Next two terms: 16 and -8 [Divide by -2 each time]	(l)	$1, \frac{1}{2}, \frac{1}{4},$ ________ , ________ . **Solution**: Next two terms: $\frac{1}{8}$ and $\frac{1}{16}$ [Divide by *2* each time]
(m)	$9, -3, 1,$ ________ , ________ . **Solution**: Next two terms: $-\frac{1}{3}$ and $\frac{1}{9}$ [Divide by -3 each time]	(n)	$1, 4, 9,$ ________ , ________ . **Solution**: Next two terms: 16 and 25 [Square the positive integers]

Exercise 1.1: Complete the following sequences

(a)	$4, 8, 12,$ ________ , ________ .	(b)	$15, 10, 5,$ ________ , ________ .
(c)	$7, 10, 13,$ ________ , ________ .	(d)	$1, 3, 9,$ ________ , ________ .
(e)	$88, 44, 22,$ ________ , ________ .	(f)	$3, -6, 12,$ ________ , ________ .
(g)	$2, 6, 18,$ ________ , ________ .	(h)	$1, -2, 4,$ ________ , ________ .
(i)	$50, 40, 30,$ ________ , ________ .	(j)	$2, 4, 8,$ ________ , ________ .
(k)	$1, 8, 27,$ ________ , ________ .	(l)	$5.5, 5.0, 4.5,$ ________ , ________ .
(m)	$0.1, -0.2, 0.3,$ ________ , ________ .	(n)	$0.9, 1.8, 2.7,$ ________ , ________ .
(o)	$\frac{1}{2}, \frac{3}{2}, \frac{5}{2},$ ________ , ________	(p)	$\frac{1}{3}, \frac{1}{6}, \frac{1}{12},$ ________ , ________
(q)	$1, \frac{3}{4}, \frac{1}{2},$ ________ , ________	(r)	$\frac{5}{4}, 1, \frac{3}{4},$ ________ , ________
(s)	$2, \frac{7}{4}, \frac{3}{2},$ ________ , ________	(t)	$\frac{2}{6}, \frac{4}{5}, \frac{6}{4},$ ________ , ________
(u)	$2\frac{1}{3}, 3\frac{1}{3}, 4\frac{1}{3},$ ________ , ________	(v)	$1\frac{1}{2}, 3, 4\frac{1}{2},$ ________ , ________
(w)	$5, 2\frac{1}{2}, \frac{5}{4},$ ________ , ________	(x)	$8\frac{1}{2}, 4\frac{1}{2}, \frac{1}{2},$ ________ , ________
(y)	$10\frac{1}{4}, 5\frac{1}{4}, \frac{1}{4},$ ________ , ________	(z)	$-1\frac{1}{4}, 1\frac{3}{4}, 4\frac{3}{4},$ ________ , ________

The Fibonacci Sequence

The **Fibonacci sequence** is a series of numbers in which each number (after the first two) is the sum of the two preceding ones. The sequence typically starts with 1, The first few terms of the Fibonacci sequence are: 1, 1, 2, 3, 5, 8, 13, 21, …

Worked Examples:

1. Complete the following Fibonacci-like sequences below:

(a) $0, 1, 1, 2, 3$ ________ , ________ .

Solution:

Next two terms: 5 and 8

(b) $2, 3, 5, 8, 13$ ________ , ________ .

Solution:

Next two terms: 21 and 34

(c) $5, 8, 13, 21, 34$ ________ , ________ .

Solution:

Next two terms: 55 and 89

(d) $1, 4, 5, 9, 14$ ________ , ________ .

Solution:

Next two terms: 23 and 37

(e) $3, 6, 9, 15, 24$ ________ , ________ .

Solution:

Next two terms: 39 and 63

(f) $2, 4, 6, 10, 16$ ________ , ________ .

Solution:

Next two terms: 26 and 42

(g) $5, 11, 16, 27, 43$ ________ , ________ .

Solution:

Next two terms: 70 and 113

(h) $7, 11, 18, 29, 47$ ________ , ________ .

Solution:

Next two terms: 76 and 123

2. If the third and fourth terms of a Fibonacci sequence are 15 and 25 respectively, find the first five terms of the sequence.

Solution:

$T_3 = 15, T_4 = 25$

$T_2 = T_4 - T_3 = 25 - 15 = 10,\ \ T_1 = T_3 - T_2 = 15 - 10 = 5$

$T_5 = T_3 + T_4 = 15 + 25 = 40$

Hence, the first 5 terms are $5, 10, 15, 25, 40$.

3. If the fifth and sixth terms of a Fibonacci sequence are 55 and 90 respectively, find the first six terms of the sequence.

Solution:

$T_5 = 55, T_6 = 90$

$T_4 = T_6 - T_5 = 90 - 55 = 35,\ \ T_3 = T_5 - T_4 = 55 - 35 = 20$

$T_2 = T_4 - T_3 = 35 - 20 = 15,\qquad T_1 = T_3 - T_2 = 20 - 15 = 5$

Hence, the first 5 terms are $5, 15, 20, 35, 55, 90$.

<u>Exercise 1.2</u>: Complete the following Fibonacci-like sequences

(a)	1, 2, 3, ________ , ________ .	(b)	3, 4, 7, ________ , ________ .
(c)	4, 7, 11, ________ , ________ .	(d)	0.2, 0.3, 0.5, ________ , ________ .
(e)	10, 15, 25, ________ , ________ .	(f)	$\frac{1}{3}, \frac{1}{2}, \frac{5}{6},$ ________ , ________
(g)	6, 9, 15, ________ , ________ .	(h)	0.1, 0.1, 0.2, ________ , ________ .
(i)	$\frac{1}{5}, \frac{2}{5}, \frac{3}{5},$ ________ , ________	(j)	2.5, 4.5, 7, ________ , ________ .
(k)	$\frac{1}{4}, \frac{1}{4}, \frac{1}{2},$ ________ , ________	(l)	2, 3.5, 5.5, ________ , ________ .
(m)	2, 4, 6, ________ , ________ .	(n)	7, 11, 18, ________ , ________ .
(o)	2.2, 3.5, 5.7, ________ , ________ .	(p)	$\frac{2}{5}, \frac{3}{5}, 1,$ ________ , ________

<u>Exercise 1.3</u>: If the second and fourth terms of a Fibonacci sequence are 18 and 47 respectively, find the first six terms of the sequence.

<u>Exercise 1.4</u>: If the third and fifth terms of a Fibonacci sequence are 60 and 160 respectively, find the first seven terms of the sequence.

Sequence of Ordered Pairs

A sequence of ordered pairs is a list of elements where each element is a pair of values arranged in a specific order. In mathematical terms, an ordered pair is represented as (x, y), where x is the first element and y is the second element.

A finite sequence of ordered pairs: $(1, 2), (3, 4), (5, 6)$.

An infinite sequence of ordered pairs: $(1, 2), (2, 4), (3, 6)$, ...

Worked Examples:

1. Complete the following sequence of ordered pairs.

(a) $(1, 2), (2, 4), (3, 6)$, ________ , ________

Solution:

The first element increases by 1 each time.

The second element is twice the first element

The next two terms are $(4, 8)$ and $(5, 10)$

(b) $(3, 9), (4, 16), (5, 25)$, ________ , ________

Solution:

The first element increases by 1 each time.

The second element is the square of the first element.

The next two terms are $(6, 36)$ and $(7, 49)$

(c) $(1, -3), (3, -9), (5, -15)$, ________ , ________ .

Solution:

The first element increases by 2 each time.

The second element is thrice the negative value of the first element

The next two terms are $(7, -21)$ and $(9, -27)$.

(d) $(1, 1), (2, 8), (3, 27)$ ________ , ________ .

Solution:

The first element increases by 3 each time.

The second element is the cube of the first element

The next two terms are $(4, 64)$ and $(5, 125)$

(e) $\left(\frac{1}{2}, \frac{1}{4}\right), \left(\frac{3}{2}, \frac{9}{4}\right), \left(\frac{5}{2}, \frac{25}{4}\right)$, ________ , ________ .

Solution:

The first element increases by 1 each time.

The second element is the square of the first element

The next two terms are $\left(\frac{7}{2}, \frac{49}{4}\right)$ and $\left(\frac{9}{2}, \frac{81}{4}\right)$.

(f) $\left(1\frac{1}{2}, 3\frac{1}{2}\right), \left(2\frac{1}{2}, 4\frac{1}{2}\right), \left(3\frac{1}{2}, 5\frac{1}{2}\right)$, ________ , ________ .

Solution:

The first element increases by 1 each time.

The second element follows the pattern

$\text{first element} \times (\text{next whole number}) + \frac{1}{4}$

The next two terms are $\left(4\frac{1}{2}, 6\frac{1}{2}\right)$ and $\left(5\frac{1}{2}, 7\frac{1}{2}\right)$.

Exercise 1.5: Write down the next two terms in each of the following sequences of ordered pairs.

(a)	$(1,2),(2,4),(3,6),$ ________ , ________ .	(b)	$(1.5,3),(2.5,5),(3.5,7),$ ________ , ________ .
(c)	$(2,1),(4,2),(6,3),$ ________ , ________ .	(d)	$\left(1\frac{1}{2},2\right),\left(2\frac{1}{2},4\right),\left(3\frac{1}{2},6\right),$ ________ , ________ .
(e)	$\left(\frac{1}{2},0.5\right),\left(\frac{3}{2},1.5\right),\left(\frac{5}{2},2.5\right),$ ________ , ________ .	(f)	$(2,-4),(4,-16),(6,-36),$ ________ , ________ .
(g)	$(-3,9),(-1,8),(1,7),$ ________ , ________ .	(h)	$(-1,1),(-0.5,0.5),(0,0),$ ________ , ________ .

Exercise 1.6: Complete the missing terms in the following sequence of ordered pairs.

(a)	$(1,3),(2,$ ______ $),($ ______ $,9),(4,12)$, $(5,$ ______ $)$.
(b)	$(1.0,0.5),($ ______ $,1),(3.0,$ ______ $),($ ______ $,2.0)$, $(5.0,2.5)$.
(c)	$($ ______ $,-2),(0,-1),($ ______ $,0),(2,$ ______ $)$, $(3,2)$.
(d)	$\left(1\frac{1}{2},3\right),\left(2\frac{1}{2},\right.$ ______ $\left.\right),($ ______ $,7),\left(4\frac{1}{2},9\right),($ ______ $,11)$
(e)	$\left(\frac{1}{3},\frac{1}{9}\right),\left(\frac{2}{3},\right.$ ______ $\left.\right),($ ______ $,1),\left(\frac{4}{3},\right.$ ______ $\left.\right),\left(\right.$ ______ $\left.,\frac{25}{9}\right)$
(f)	$(-4,16),($ ______ $,9),(-2,$ ______ $),(0,$ ______ $),($ ______ $,0)$
(g)	$(1,-2),(2,-1),(3,$ ______ $),($ ______ $,1)$, $(5,2)$.
(h)	$(2,4),($ ______ $,6),(4,$ ______ $),(6,12)$, $(8,$ ______ $)$.
(i)	$(0.5,0.25),(1.0,$ ______ $),($ ______ $,2.25),(2.0,$ ______ $)$, $(2.5,6.25)$.
(j)	$(3.0,9.0),(2.5,$ ______ $),(2.0,4.0),($ ______ $,2.25)$, $(1.0,$ ______ $)$.

UNIT 2 : INDICES

Learning Objectives

By the end of this chapter, you should be able to

- Identify negative powers: $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n, n \in \mathbb{Z}^+$.
- Recognise powers of $\frac{1}{2}$ and $\frac{1}{3}$ as square root and cube root respectively.
- Work with indices in the form $\left(\frac{a}{b}\right)^n$ and $(ab)^n$ where $n \geq 0$ and $a, b \in \mathbb{Z}^+$
- Solve problems involving unit fractional indices in the form $\frac{1}{p}$ where $p \in \mathbb{Z}^+$.
- Solve problems involving perfect squares, square roots, cubes and cube roots.

Negative Indices

Negative Indices

When attempting questions on indices, some answers will lead to a **negative power**. The answers need to be **expressed as positive powers only**. In order to do so, the following formulae are used:

$$a^{-n} = \frac{1}{a^n}$$

$$\frac{1}{a^{-n}} = a^n$$

Notes: In the above formulae:

(i) $a \neq 0$,

(ii) a^{-n} is called the reciprocal of a^n.

For examples: (*Expressing as positive powers*)

(a) $a^{-1} = \frac{1}{a^3}$, (b) $y^{-2} = \frac{1}{y^2}$ (c) $3x^{-3} = \frac{3}{x^3}$

(d) $\frac{1}{y^{-5}} = y^5$ (e) $\frac{1}{2a^{-8}} = \frac{a^8}{2}$ (f) $\frac{3}{4y^{-5}} = \frac{3y^5}{4}$

Exercise 2.1: Express the following in terms of positive powers

(a)	x^{-2}	(b)	y^{-6}	(c)	c^{-8}
(d)	$5a^{-7}$	(e)	$8e^{-9}$	(f)	$12r^{-5}$

Exercise 2.2: Express the following in terms of **positive** powers

(a)	$\frac{1}{w^{-4}}$	(b)	$\frac{1}{m^{-11}}$	(c)	$\frac{1}{5n^{-6}}$
(d)	$\frac{1}{8p^{-12}}$	(e)	$\frac{7}{9q^{-8}}$	(f)	$\frac{6}{13r^{-10}}$

More Examples: (*Expressing as negative powers*)

(a) $b^2 = \frac{1}{b^{-2}}$, (b) $\frac{1}{y^3} = y^{-3}$ (c) $\frac{5}{x^6} = 5x^{-6}$

(d) $\frac{1}{3a^4} = \frac{a^{-4}}{3}$ (e) $\frac{2}{7x^5} = \frac{2x^{-5}}{7}$ (f) $\frac{8}{a^3b^6} = 8a^{-3}b^{-6}$

Exercise 2.3: Express the following in terms of **negative** powers

(a)	c^3	(b)	$\frac{1}{m^4}$	(c)	$\frac{9}{y^5}$
(d)	$\frac{1}{2r^9}$	(e)	$\frac{2}{5s^6}$	(f)	$\frac{4}{x^{10}y^7}$

More Examples: *Evaluate*

(a) $2^{-1} = \frac{1}{2^1} = \frac{1}{2}$, (b) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$, (c) $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

(d) $\frac{1}{2^{-3}} = 2^3 = 8$ (e) $\frac{1}{3^{-4}} = 3^4 = 81$ (f) $\frac{2}{4^{-3}} = 2 \times 4^3 = 2 \times 64 = 128$

Exercise 2.4: Evaluate

(a)	4^{-1}	(b)	7^{-1}	(c)	5^{-2}
(d)	6^{-2}	(e)	2^{-3}	(f)	3^{-3}
(g)	$\frac{1}{4^{-3}}$	(h)	$\frac{1}{6^{-3}}$	(i)	$\frac{1}{2^{-4}}$
(j)	$\frac{1}{5^{-4}}$	(k)	$\frac{3}{4^{-2}}$	(l)	$\frac{5}{3^{-3}}$

Indices of the form $(ab)^n$

For $n \geq 0$ and $a, b \in \mathbb{Z}^+$,

$$(ab)^n = (a \times b)^n = a^n \times b^n = a^n b^n$$

For examples:

(a) $(mn)^4 = m^4 n^4$,
(b) $(a^3 b)^3 = a^8 b^3$
(c) $(x^2 y^5)^2 = x^4 y^{10}$

(d) $(3b)^4 = 3^4 b^4 = 81b^4$
(e) $(2x^2 y^3)^5 = 2^5 x^{10} y^{15}$ $= 32x^{10} y^{15}$
(f) $4(8a)^2 = 4 \times 64a^2 = 256a^2$

Exercise 2.5: Simplify

(a)	$(pq)^3$	(b)	$(abc)^2$	(c)	$(x^2 y^3)^2$
(d)	$(m^3 n^4)^2$	(e)	$(a^3 b^2)^2$	(f)	$(p^2 q^4)^3$
(g)	$(4y)^3$	(h)	$(5c)^2$	(i)	$(3x^2 z)^4$
(j)	$(5a^2 b^3)^2$	(k)	$3(2x)^3$	(l)	$2(5y)^2$

Indices of the form $\left(\frac{a}{b}\right)^n$

For $n \geq 0$ and $a, b \in \mathbb{Z}^+$,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

For examples:

(a) $\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$, (b) $\left(\frac{3}{4}\right)^3 = \frac{3^3}{4^3} = \frac{27}{64}$ (c) $\left(\frac{a^5}{b^3}\right)^2 = \frac{(a^5)^2}{(b^3)^2} = \frac{a^{10}}{b^6}$

(d) $\left(\frac{2x^3}{y^2}\right)^3 = \frac{2^3(x^3)^3}{(y^2)^3} = \frac{8x^9}{y^6}$ (e) $\left(\frac{3m^2}{4n^4}\right)^2 = \frac{3^2(m^2)^2}{4^2(n^4)^2} = \frac{9m^4}{16n^8}$ (f) $\left(\frac{3m^2}{4n^4}\right)^2 = \frac{3^2(m^2)^2}{4^2(n^4)^2} = \frac{9m^4}{16n^8}$

<u>Exercise 2.6</u>: Simplify and evaluate where possible

(a)	$\left(\frac{3}{4}\right)^2$	(b)	$\left(\frac{5}{2}\right)^3$	(c)	$\left(\frac{2}{5}\right)^3$
(d)	$\left(\frac{4}{7}\right)^2$	(e)	$\left(\frac{b^3}{c^2}\right)^2$	(f)	$\left(\frac{p^4}{q^5}\right)^3$
(g)	$\left(\frac{3x^2}{y^4}\right)^3$	(h)	$\left(\frac{2a^3}{b^2}\right)^4$	(i)	$\left(\frac{5a^2}{3b^4}\right)^2$
(j)	$\left(\frac{4x^3}{2y^2}\right)^3$	(k)	$\left(\frac{2a^2b}{3c^3}\right)^3$	(l)	$\left(\frac{4m^2}{5n^4}\right)^2$

Fractional Indices

A **fractional index number** consists of a base and a fractional power. Some examples are:

$$4^{\frac{1}{2}}, 8^{\frac{1}{3}}, 16^{\frac{3}{2}}, 125^{\frac{1}{3}}, \ldots$$

In general,

(i) $a^{\frac{1}{n}}$ is called the nth root of a and $a^{\frac{1}{n}} = \sqrt[n]{a}$

For examples:

$$a^{\frac{1}{2}} = \sqrt{a} \text{ (also called as the square root of } a)$$
$$a^{\frac{1}{3}} = \sqrt[3]{a} \text{ (also called as the cube root of } a)$$
$$a^{\frac{1}{4}} = \sqrt[4]{a} \text{ (also called as the fourth root of } a)$$
$$a^{\frac{1}{5}} = \sqrt[5]{a} \text{ (also called as the fifth root of } a)$$

(ii) $a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} = \left(a^{\frac{1}{n}}\right)^m$ is called the nth root of a raised to the power of m.

For examples:

$$a^{\frac{3}{2}} = (a^3)^{\frac{1}{2}} = \sqrt{a^3}$$
$$a^{\frac{3}{4}} = (a^3)^{\frac{1}{4}} = \sqrt[4]{a^3}$$
$$a^{\frac{2}{5}} = (a^2)^{\frac{1}{5}} = \sqrt[5]{a^2}$$

Some further examples:

(i) $9^{\frac{1}{2}} = (3^2)^{\frac{1}{2}} = 3^{2\times\frac{1}{2}} = 3^1 = 3$ or $9^{\frac{1}{2}} = \sqrt{9} = 3$

(ii) $8^{\frac{1}{3}} = (2^3)^{\frac{1}{3}} = 2^{3\times\frac{1}{3}} = 2^1 = 2$ or $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

(iii) $16^{\frac{1}{4}} = (2^4)^{\frac{1}{4}} = 2^{4\times\frac{1}{4}} = 2^1 = 2$ or $16^{\frac{1}{4}} = \sqrt[4]{16} = 2$

(iv) $25^{0.5} = 25^{\frac{1}{2}} = (5^2)^{\frac{1}{2}} = 5^{2\times\frac{1}{2}} = 5^1 = 5$

(v) $8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^{3\times\frac{2}{3}} = 2^2 = 4$

(vi) $16^{\frac{3}{2}} = (4^2)^{\frac{3}{2}} = 4^{2\times\frac{3}{2}} = 4^3 = 64$

Exercise 2.7: Evaluate

(a) $8^{\frac{1}{3}}$	(b) $9^{\frac{1}{2}}$	(c) $64^{\frac{1}{2}}$
(d) $125^{\frac{1}{3}}$	(e) $27^{\frac{2}{3}}$	(f) $49^{\frac{3}{2}}$

Fractional Indices with Negative Powers

For examples:

(a) $(2^3)^{-\frac{2}{3}} = 2^{3\times-\frac{2}{3}} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$.

(b) $(4^{-2})^{\frac{3}{2}} = 4^{-2\times\frac{3}{2}} = 4^{-3} = \frac{1}{4^3} = \frac{1}{64}$.

(c) $\left(1\frac{11}{25}\right)^{-\frac{1}{2}} = \left(\frac{36}{25}\right)^{-\frac{1}{2}} = \left(\frac{25}{36}\right)^{\frac{1}{2}} = \frac{25^{\frac{1}{2}}}{36^{\frac{1}{2}}} = \frac{5}{6}$

<u>Exercise 2.8</u>: Evaluate

(a) $(3^3)^{-\frac{2}{3}}$	(b) $(2^5)^{-\frac{2}{5}}$	(c) $(7^2)^{-\frac{1}{2}}$
(d) $(8^3)^{-\frac{1}{3}}$	(e) $(4^5)^{-\frac{3}{5}}$	(f) $(5^{-6})^{\frac{1}{2}}$
(g) $(6^{-3})^{\frac{2}{3}}$	(h) $\left(1\frac{7}{9}\right)^{-\frac{1}{2}}$	(i) $\left(2\frac{10}{27}\right)^{-\frac{1}{3}}$

Square Numbers and Square Roots

A **square number**, also known as a ***perfect square***, is an integer that can be expressed as the square of another integer. For example, 16 is a square number because it can be written as 4^2, where 4 is also an integer.

The **square root** of a number, on the other hand, is a value that, when multiplied by itself, gives the original number. For instance, the square root of 25 is 5 because $5 \times 5 = 25$. Square roots can be calculated for any ***non-negative number***, not just perfect squares. If a number is not a perfect square, its square root will typically be an **irrational number** (*a number that cannot be expressed as a simple fraction*). The symbol $\sqrt{\ }$ is used to denote the square root.

The square root in index form

The square of any positive number x can be written as $\sqrt{x}$ or $x^{\frac{1}{2}}$. So $\sqrt{x} = x^{\frac{1}{2}}$.

For examples: Find the square root of

(a) 9

Solution:

Method 1:-

$\sqrt{9} = \sqrt{3 \times 3} = 3$

Method 2:-

$\sqrt{9} = 9^{\frac{1}{2}} = (3^2)^{\frac{1}{2}}$

$= 3^{2\times\frac{1}{2}}$

$= 3$

(b) 64

Solution:

Method 1:-

$\sqrt{64} = \sqrt{8 \times 8} = 8$

Method 2:-

$\sqrt{64} = 64^{\frac{1}{2}} = (8^2)^{\frac{1}{2}}$

$= 8^{2\times\frac{1}{2}}$

$= 8$

Exercise 2.9: Find the square root of

(a) 1	(b) 4	(c) 16
(d) 36	(e) 49	(f) 81
(g) 100	(h) 121	(i) 144

Harder Examples using Prime Factorisation:
Find the square root of
(a) 225

Solution:

$$\sqrt{225} = \sqrt{\underbrace{3 \times 3}_{} \times \underbrace{5 \times 5}_{}}$$
$$= 3 \times 5$$
$$= 15$$

3	225
3	75
5	25
5	5
	1

(b) 576

Solution:

$$\sqrt{576} = \sqrt{\underbrace{2 \times 2}_{} \times \underbrace{2 \times 2}_{} \times \underbrace{2 \times 2}_{} \times \underbrace{3 \times 3}_{}}$$
$$= 2 \times 2 \times 2 \times 3$$
$$= 24$$

2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

Exercise 2.10: Using prime factorisation to find the square root of

(a) 256	(b) 324	(c) 400
(d) 484	(e) 676	(f) 784

Exercise 2.11: Using prime factorisation to find the square root of

(a) 900	(b) 1764	(c) 2304
(d) 3136	(e) 3600	(f) 4096
(g) 5184	(h) 6400	(i) 10 000